Econ 262A: Problem Set 1

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1. Instrumental Variables Simulation

Simulate a data set with 10,000 observations (in your choice of program), and construct the following variables:

$$X_i \sim N(0,1)$$

$$Z_i = \begin{cases} 0 & \text{with probability } 0.5 \\ 1 & \text{with probability } 0.5 \end{cases}$$

$$\begin{pmatrix} u_i \\ \varepsilon_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.25 \\ 0.25 & 1 \end{pmatrix} \right)$$

$$T_i = \mathbb{I}\left[-0.5 + 0.25X_i + Z_i + u_i \geq 0\right]$$

$$Y_i = 0.5X_i + T_i + \varepsilon_i$$

As an example, imagine that we are studying the returns to college: Y is individual earnings, T is college or not, X is a personal characteristic, and Z is an instrument for college completion.

1.1

Report in a table the following regressions:

1. OLS regression of Y on X and T
2. OLS regression of T on Z and X
3. IV regression of Y on T and X with Z as instrument for T

The output is shown in Table 1. After instrumenting for T with Z, we obtain an estimate on the treatment effect close to 1, while the coefficient in the first OLS was biased due to T being endogenous. IV should work here in estimating the ATE on compliers (which is same for everyone) because the instrument was randomly assigned, the instrument directly affects treatment and not individual earnings outcome, and the instrument satisfies monotonicity.

1.2

Redo the simulation, but let the coefficient on T in the equation for Y be α_i, which will vary with the error terms:

$$\begin{pmatrix} \alpha_i \\ u_i \\ \varepsilon_i \end{pmatrix} \sim N\left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.25 & 0.25 \\ 0.25 & 1 & 0.25 \\ 0.25 & 0.25 & 1 \end{pmatrix} \right)$$

In the returns to college example, what does α_i being correlated with ε_i mean? What does α_i being correlated with u_i mean?

Table 1: Problem 1.1

	Dependent variable:		
	Y	T	Y
	OLS	*OLS*	*instrumental variable*
	OLS	First Stage	IV
	(1)	(2)	(3)
Z		0.386***	
		(0.009)	
X	0.467***	0.096***	0.505***
	(0.010)	(0.004)	(0.011)
T	1.347***		0.963***
	(0.020)		(0.052)
Constant	−0.179***	0.300***	0.010
	(0.014)	(0.006)	(0.028)
Observations	10,000	10,000	10,000
R^2	0.449	0.188	0.429
Note:			*p<0.1; **p<0.05; ***p<0.01

α_i being correlated with ε_i means that the effect of college on an individual's earnings is related to unobserved determinants of that individual's earnings. Since they are positively correlated, individuals with unobserved earnings ability also tend to get more out of college. α_i being correlated with u_i means that the effect of college on an individual's earnings is related to unobservable factors on how likely that individual selects into college. Again, since they are positively correlated, people who are more likely to attend college also tend to get bigger earnings benefits from college.

Rerun the table from figure 1.1.

Table 2: Problem 1.2

	Dependent variable:		
	Y	T	Y
	OLS	*OLS*	*instrumental variable*
	OLS	First Stage	IV
	(1)	(2)	(3)
Z		0.381***	
		(0.009)	
X	0.440***	0.083***	0.486***
	(0.013)	(0.005)	(0.014)
T	1.543***		1.011***
	(0.026)		(0.069)
Constant	−0.182***	0.309***	0.083**
	(0.018)	(0.006)	(0.037)
Observations	10,000	10,000	10,000
R^2	0.353	0.175	0.326
Note:			*p<0.1; **p<0.05; ***p<0.01

The output is given in Table 2. Treatment effects are now heterogenous, but we still see a LATE of about 1.

Using the approach from Imbens and Rubin (1997), plot the distributions of Y_0 and Y_1 for the compliers and the distribution of Y for the always takers and Y for the never takers. Also use this approach to plot the distributions of X for the always takers, never takers, and compliers.

The distributions of Y for the always-takers and never-takers and the distributions of Y_0 and Y_1 for the compliers are shown in Figure 1. The distributions of X for all 3 types are shown in Figure 2.

What is your interpretation for the difference in the distribution of Y_0 for compliers and Y for the never takers? What is your interpretation of the difference in distributions of X for compliers and never takers?

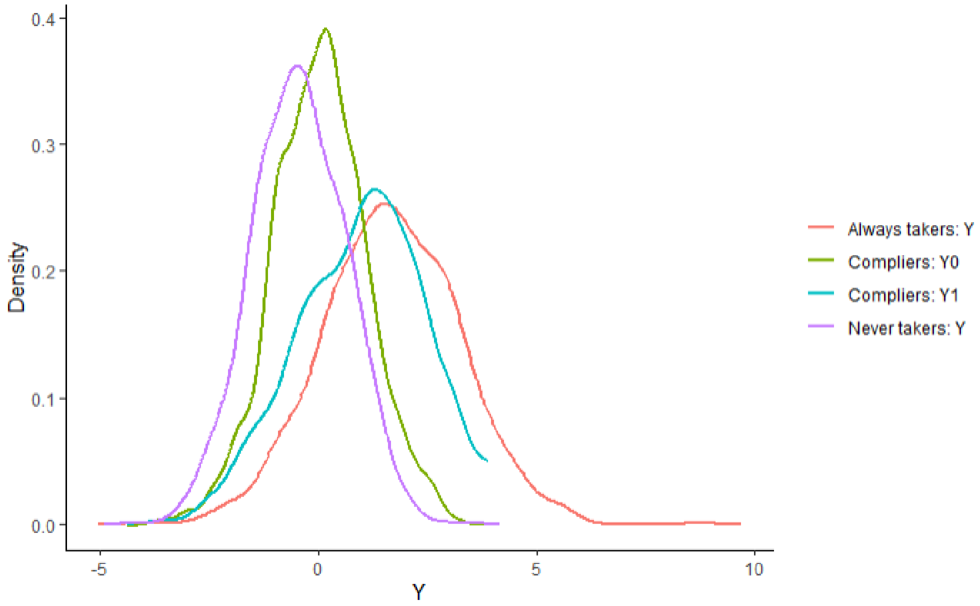

Figure 1: Distribution of Y_0 and Y_1 for Compliers, and Y for Always-Takers and Never-Takers

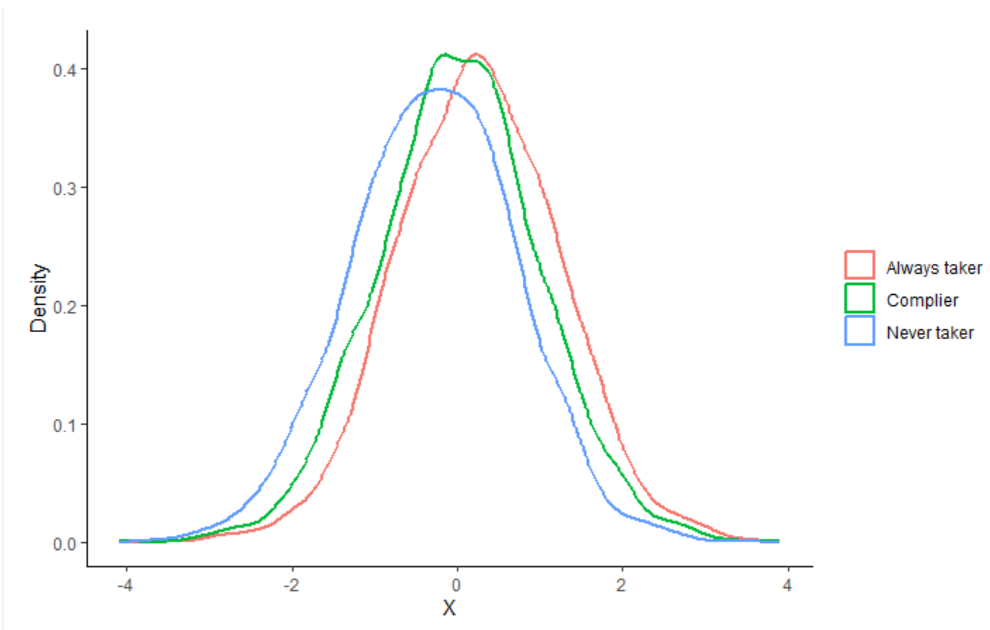

Figure 2: Distribution of X for Compliers, Always-takers, and Never-takers

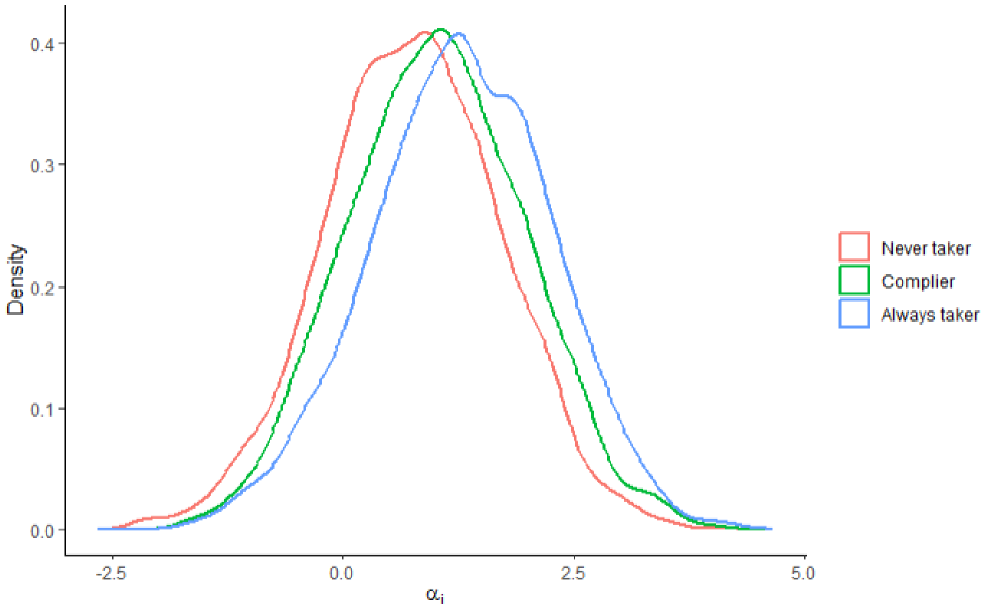

Figure 3: Distribution of Treatment Effects Across Never takers, compliers, and always takers.

The distribution of Y_0 for compliers is shifted slightly to the right of the distribution of Y for never-takers. Thus, never-takers experience worse earnings outcomes compared to compliers, even in the absence of treatment. This reflects the inherent difference between the two-types' propensity to select into treatment, where compliers are able to be pushed into college via the instrument, due to having a higher baseline outcome potential.

The differences in the distributions of X reflect the three types individual characteristics, where higher values of X raises the likelihood an individual will select into treatment. Thus, since the never-taker distribution is shifted to the left of the complier distribution, compliers are more likely to select into treatment/college education, even before presence of the instrument.

While we do not observe this in any real-world data, plot the distributions of α_i for the never takers, compliers, and always takers. What is your interpretation of the difference in distributions across types?

The distributions are plotted in Figure 3. As we can see, the distribution for the never takers is the lowest, followed by the compliers, followed by the always takers. This means that always-takers experience the greatest earnings returns on college, followed by compliers, and then never takers. The reason we see this difference is because the treatment effect is positively correlated with unobservable factors in treatment selection, namely, u_i. In other words, those who are more likely to select into college treatment also experience greater earnings returns from it.

2. Accounting for Defiers

Suppose you have a binary instrument, $Z \in \{0, 1\}$, treatment, $D \in \{0, 1\}$, and outcome, $Y \in \{0, 1\}$.

(a)

Prove the following relationship between the IV estimand and the treatment effects for compliers and defiers:

$$E[Y_i(1)-Y_i(0) \mid i \text{ is a complier}] = \frac{1}{1+\lambda}\frac{E[Y_i(1,D_i(1)) - Y_i(0,D_i(0))]}{E[D_i(1)-D_i(0)]} + \frac{\lambda}{1+\lambda} E[Y_i(1)-Y_i(0) \mid i \text{ is a defier}]$$

where

$$\lambda = \frac{\Pr(i \text{ is a defier})}{\Pr(i \text{ is a complier}) - \Pr(i \text{ is a defier})}.$$

Proof. We can write (by exclusion restriction) the causal effect of the instrument on Y

$$Y_i(1,D_i(1)) - Y_i(0,D_i(0)) = (Y_i(1)-Y_i(0))(D_i(1)-D_i(0)),$$

and then splitting into types (where always-takers and never-takers are 0 in their first-stage), by the tower rule, we have

$$\begin{aligned}
E[(Y_i(1,D_i(1)) - Y_i(0,D_i(0))] &= E[Y_i(1)-Y_i(0) \mid D_i(1)-D_i(0)=1]\Pr(D_i(1)-D_i(0)=1) \\
&\quad - E[Y_i(1)-Y_i(0) \mid D_i(1)-D_i(0)=-1]\Pr(D_i(1)-D_i(0)=-1) \\
&= E[Y_i(1)-Y_i(0) \mid i \text{ is a complier}]\Pr(i \text{ is a complier}) \\
&\quad - E[Y_i(1)-Y_i(0) \mid i \text{ is a defier}]\Pr(i \text{ is a defier}).
\end{aligned}$$

By the same logic, we can write the first-stage effect as

$$E[D_i(1)-D_i(0)] = \Pr(i \text{ is a complier}) - \Pr(i \text{ is a defier}).$$

Thus, we can write the IV estimand as

$$\begin{aligned}
\beta^{IV} &= \frac{E[Y_i(1,D_i(1)) - Y_i(0,D_i(0))]}{E[D_i(1)-D_i(0)]} \\
&= \frac{E[Y_i(1)-Y_i(0) \mid i \text{ is a complier}]\Pr(i \text{ is a complier}) - E[Y_i(1)-Y_i(0) \mid i \text{ is a defier}]\Pr(i \text{ is a defier})}{\Pr(i \text{ is a complier}) - \Pr(i \text{ is a defier})},
\end{aligned}$$

and rearranging,

$$\begin{aligned}
&E[Y_i(1)-Y_i(0) \mid i \text{ is a complier}] \\
&\quad = \beta^{IV}\frac{\Pr(i \text{ is a complier}) - \Pr(i \text{ is a defier})}{\Pr(i \text{ is a complier})} + E[Y_i(1)-Y_i(0) \mid i \text{ is a defier}]\frac{\Pr(i \text{ is a defier})}{\Pr(i \text{ is a complier})} \\
&\quad = \beta^{IV}\frac{1}{1+\lambda} + E[Y_i(1)-Y_i(0) \mid i \text{ is a defier}]\frac{\lambda}{1+\lambda}
\end{aligned}$$

by the definition of λ. □

(b)

Prove that the share of defiers is bounded by the share of people who have treatment off among those with instrument on, $(D = 0 \mid Z = 1)$, and the share of people who have treatment on among those with instrument off, $(D = 1 \mid Z = 0)$:

$$\Pr(i \text{ is a defier}) \leq \min\{\Pr(D = 0 \mid Z = 1), \Pr(D = 1 \mid Z = 0)\}.$$

Proof. Those with treatment off and instrument on are either defiers or never-takers. Similarly, those with treatment on and instrument off are either defiers or always-takers. Denote

$$\Pr(i \text{ is a defier}) = \phi_d$$
$$\Pr(i \text{ is an always-taker}) = \phi_a$$
$$\Pr(i \text{ is a never-taker}) = \phi_n.$$

Thus, we can write

$$\Pr(D = 0 \mid Z = 1) = \phi_d + \phi_n \geq \phi_d$$

and

$$\Pr(D = 1 \mid Z = 0) = \phi_d + \phi_a \geq \phi_d,$$

since $\phi_a, \phi_n \geq 0$. Thus, ϕ_d is weakly less than the minimum. □

We will now use these two formulas to consider the following empirical problem. Suppose you have the following values for your data:

$$\Pr(Z = 0, D = 0) = 0.3$$
$$\Pr(Z = 0, D = 1) = 0.1$$
$$\Pr(Z = 1, D = 0) = 0.4$$
$$\Pr(Z = 1, D = 1) = 0.2$$
$$\text{IV estimate} = 0.5$$

(c)

If you had no defiers, what share of individuals are compliers?

With no defiers, we can identify the share of always-takers and never-takers:

$$\Pr(D = 1 \mid Z = 0) = \frac{0.1}{0.1 + 0.3} = 0.25 = \phi_a$$

$$\Pr(D = 0 \mid Z = 1) = \frac{0.4}{0.4 + 0.2} = 0.67 = \phi_d$$

and so $\phi_c = 1 - \phi_a - \phi_n = 0.08$.

(d)

Making no assumptions about defiers, what is the maximum share of individuals who could be defiers?

Using the bound in (b),

$$\phi_d \leq \min\{\Pr(D = 1 \mid Z = 0), \Pr(D = 0 \mid Z = 1)\} = 0.25.$$

(e)

Suppose that you have the maximum number of defiers. What is the share of compliers?

With $\phi_d = 0.25$, we have

$$\phi_a = \Pr(D = 1 \mid Z = 0) - \phi_d = 0,$$

$$\phi_d = \Pr(D = 0 \mid Z = 1) - \phi_d = 0.42,$$

and so $\phi_c = 1 - \phi_d - \phi_n - \phi_a = 0.33$.

(f)

Still assuming you have a maximum number of defiers, the equation above for part (a) says the IV estimand no longer gives you the LATE for the compliers. We know, however, that the treatment effect for defiers (and everybody) is bounded between -1 and 1 because of the range for Y. What is the range of possible values for the complier LATE that are consistent with the observed IV estimand and the set of possible values for the defier LATE?

Plugging in, we find $\lambda = \frac{0.25}{0.33-0.25} = 3$. Thus, plugging in λ and the IV estimate into (1), we get:

$$LATE_{\text{complier}} = 0.125 + 0.75 LATE_{\text{defier}}.$$

Plugging in the bounds on the LATE for defiers of $-1, 1$, we bound the complier LATE

$$-0.625 \leq LATE_{\text{complier}} \leq 0.875.$$

3. Testing IV Assumptions and Bounding Treatment Effects

Suppose you have a binary instrument $Z_i \in \{0, 1\}$, a binary treatment $D_i \in \{0, 1\}$, and a binary outcome $Y_i \in \{0, 1\}$, with the following observed distribution:

$$P(D_i = 1 \mid Z_i = 0) = 0.25$$

$$P(D_i = 1 \mid Z_i = 1) = 0.30$$

$$E(Y_i \mid Z_i = 0, D_i = 0) = 0.20$$

$$E(Y_i \mid Z_i = 0, D_i = 1) = 0.30$$

$$E(Y_i \mid Z_i = 1, D_i = 0) = 0.40$$

$$E(Y_i \mid Z_i = 1, D_i = 1) = 0.60$$

1. Compliance types and the LATE

   (a) What is the LATE for compliers?

   (b) Assuming there are no defiers, what share of the population consists of compliers?

   (c) Assuming there are no defiers, what share of the population consists of always takers?

   We have that

   $$\phi_a = \Pr(D_i = 1 \mid Z_i = 0) = 0.25, \quad \phi_n = P(D_i = 0 \mid Z_i = 1) = 1 - P(D_i = 1 \mid Z_i = 1) = 0.70$$

   and so $\phi_c = 0.05$. For the reduced-form, we have by conditional probability

   $$\begin{aligned} E[Y_i(1, D_i(1)) - Y_i(0, D_i(0))] &= E[Y_i \mid Z_i = 1, D_i = 0] \Pr(D_i = 0 \mid Z_i = 1) \\ &\quad + E[Y_i \mid Z_i = 1, D_i = 1] \Pr(D_i = 1 \mid Z_i = 1) \\ &\quad - E[Y_i \mid Z_i = 0, D_i = 0] \Pr(D_i = 0 \mid Z_i = 0) \\ &\quad - E[Y_i \mid Z_i = 0, D_i = 1] \Pr(D_i = 1 \mid Z_i = 0) \\ &= 0.4 \cdot 0.7 + 0.6 \cdot 0.3 - 0.2 \cdot 0.75 - 0.3 \cdot 0.25 \\ &= 0.235, \end{aligned}$$

   and for the first-stage, we have

   $$E[D_i(1) - D_i(0)] = \phi_c - \phi_d = 0.05,$$

   and so

   $$\beta^{IV} = \frac{0.235}{0.05} = 4.7.$$

2. Testing instrument validity

   (a) Apply the Huber and Mellace (2015) test for instrument validity. Is the test satisfied?

   For the always-takers, we first compute the proportion of always-takers among those with $\{D = 1, Z = 1\}$: $q = \frac{\phi_a}{\phi_a + \phi_c} = 0.833$. Since $E[Y_i \mid Z = 1, D = 1] = 0.6$ and the outcome is binary (an indicator variable), we have that

   $$\Pr(Y_i = 1 \mid Z = 1, D = 1) = 0.6, \quad \Pr(Y_i = 0 \mid Z = 1, D = 1) = 0.4.$$

   For the lower bound, we first take the lower 0.833 proportion of the outcomes under $\{D = 1, Z = 1\}$, renormalize, and compute the expectation:

   $$E[Y_i = 1 \mid Z = 1, D = 1, Y \leq y^{0.833}] = \Pr(Y_i = 1 \mid Z = 1, D = 1, Y \leq y^{0.833}) = \frac{0.833 - 0.4}{0.833} = 0.52.$$

   For the upper bound, we take the upper 0.833 proportion of outcomes under $\{D = 1, Z = 1\}$, and do the same:

   $$E[Y_i = 1 \mid Z = 1, D = 1, Y \geq y^{1-0.833}] = \Pr(Y_i = 1 \mid Z = 1, D = 1, \geq y^{1-0.833}) = \frac{0.6}{0.833} = 0.72.$$

The expected outcome among those with $\{D=1, Z=0\}$ (guaranteed always-takers) should lie within $[0.52, 0.72]$ if the IV assumptions hold. However, $E(Y_i \mid Z_i = 0, D_i = 1) = 0.30 \notin [0.52, 0.72]$, so the test fails. We can repeat the process for the never-takers:

$$r = \frac{\phi_n}{\phi_n + \phi_c} = 0.933,$$

and so we get the bounds

$$E[Y_i = 1 \mid Z = 0, D = 0, Y \leq y^{0.933}] = \Pr(Y_i = 1 \mid Z = 0, D = 0, Y \leq y^{0.933}) = \frac{0.933 - 0.8}{0.933} = 0.1429,$$
$$E[Y_i = 1 \mid Z = 0, D = 0, Y \geq y^{1-0.933}] = \Pr(Y_i = 1 \mid Z = 0, D = 0, \geq y^{1-0.933}) = \frac{0.2}{0.933} = 0.2143.$$

The expected outcome for those with $\{D=0, Z=1\}$ (guaranteed never-takers) is 0.4, which does not lie within these bounds, so the test again fails. Thus, the data are inconsistent with IV assumptions.

3. Bounding the direct effect of the instrument

   (a) Suppose you want to identify the direct effect of Z_i on Y_i; that is, suppose the exclusion restriction fails, so that Z_i may affect Y_i through channels other than D_i. Use the Lee (2009) approach to construct bounds on the direct effect of Z_i on Y_i.

We can do something similar to sample selection (S) induced by a treatment (D) and apply it to treatment selection (D, which affects outcome) induced by the instrument (Z). Then, we can trim off individuals who would have been treated even without the instrument to get comparable groups and compute bounds on the direct effect of the instrument Z on Y. We first see

$$\Pr(D = 1 \mid Z = 0) = 0.25, \quad \Pr(D = 1 \mid Z = 1) = 0.30$$

so among the treated ($D = 1$) that had instrument on ($Z = 1$), a proportion $p = \frac{0.30-0.25}{0.30} = 1/6$ are those on the margin induced by the instrument. Next, we aim to compare the treated outcomes between individuals instrumented and those not:

$$E[Y \mid D = 1, Z = 1] = 0.6, \quad E[Y \mid D = 1, Z = 0] = 0.3,$$

but we have to trim $p = 1/6$ from the $\{D = 1, Z = 1\}$ population. The outcome is binary, so for the upper bound, we trim the lower 1/6 and get:

$$E[Y \mid D = 1, Z = 1, Y \geq y_{1/6}] = \frac{0.6}{5/6} = 0.72.$$

For the lower bound, we trim the upper 1/6 of outcomes and get:

$$E[Y \mid D = 1, Z = 1, Y \leq y_{5/6}] = \frac{5/6 - 0.4}{5/6} = 0.52.$$

Subtracting the average outcome under the treated group-instrument off group, we compute the Lee bounds on the direct effect of Z on Y: $[0.22, 0.42]$. Indeed, Z has a positive direct effect on Y.

4. Instrumental Variables Data Analysis

In this problem, you will analyze the effect of family size on parents' labor supply. You will use data, posted on the BruinLearn course page, similar to the data used in Angrist and Evans (1998), "Children and Their Parents' Labor Supply: Evidence from Exogenous Variation in Family Size."

The goal is to estimate the effect of having three or more children, relative to having exactly two children. Let the treatment indicator be $D_i \in \{0, 1\}$, where $D_i = 1$ indicates having three or more children. The outcomes of interest, denoted Y_i, are maternal labor supply and maternal labor income.

You will instrument for D_i using whether the parents' first two children are of the same sex, denoted $Z_i \in \{0, 1\}$. The logic of this instrument is that some parents prefer to have children of both sexes, and therefore may be more likely to have an additional child if their first two children are of the same sex.

You may complete this problem using any programming language you like (e.g. Stata or R). Your tables and figures do not need to be publication-level quality; simple, clearly labeled output is sufficient.

1. Compliance types

   (a) For the overall sample, what share of respondents are always takers, never takers, and compliers?

   We construct $D = \mathbb{I}[kidcount \geq 3], Z = \mathbb{I}[boy1st == boy2nd]$. Assuming no defiers, we compute:

   $$\phi_a = 0.356, \quad \phi_n = 0.590, \quad \phi_c = 0.054.$$

2. IV estimates of family size

   (a) Present a table reporting your IV estimates of the effect of having three or more children on:

      i. whether the mother works;

      ii. maternal labor income.

   (b) For maternal labor income, code non-working mothers as having labor income equal to 0.

   (c) For each outcome, report two specifications:

      i. a specification without controls;

      ii. a specification with controls for maternal and paternal race, education, age, and age at birth of first child.

   (d) How do you interpret the estimated coefficients?

   (e) How do you interpret the effect of including covariates on your estimates?

Table 3: IV Estimates of the Effect of ≥ 3 Children (D) on Mother Labor Supply and Income

	Dependent variable:			
	Mother Works		Income	
	(1)	(2)	(3)	(4)
D	-0.105^{***}	-0.098^{***}	$-1{,}684.388^{***}$	$-1{,}586.398^{***}$
	(0.023)	(0.023)	(543.518)	(512.430)
Constant	0.614^{***}	0.361^{***}	$8{,}314.309^{***}$	$-6{,}173.766^{***}$
	(0.009)	(0.011)	(208.840)	(241.596)
Controls	No	Yes	No	Yes
Observations	614,343	526,447	614,343	526,447
R^2	0.008	0.063	0.008	0.086
Note:				*p<0.1; **p<0.05; ***p<0.01

The estimates are shown in Table 3. We interpret the coefficient on D as the causal effect of having 3 or more children for compliers (families who are more likely to have 3 or more children if their first 2 children are same gender). In particular, for the work regression (specification 1), we estimate that having a third child reduces the probability a complier mother works by 0.105 on average, and for the income regression (specification 3), we estimate that having a third child reduces the income of complier mothers by \$1684.39 on average. If the estimates are similar with and without controls, that suggests that the chosen instrument is close to independent from individual characteristics. If the estimates change significantly, the instrument may be invalid. The IV estimates become slightly smaller in magnitude with controls. This suggests that some of the variation in mother labor participation and income associated with having a 3rd child is related to observable parental characteristics (race, education, age, and age at first birth).

3. Characteristics of compliance types

   (a) Using the Imbens and Rubin (1997) and Abadie (2003) approach, calculate the average values of the following characteristics for never takers, always takers, and compliers:

      i. maternal non-white;

      ii. years of education;

      iii. age at first birth.

   (b) What is your interpretation of the differences in demographics across these groups?

The average values are shown in Table 4. Always-takers are more likely to be non-white, have fewer years of education, and had their first birth at a year age than both compliers and never-takers. Never-takers tend to have the most years of education and are generally older at the time of their first-birth, indicating a lower willingness to have a third child. Compliers

Table 4: Average Characteristics by Compliance Type

Type	Maternal non-white	Years of education	Age at first birth
Always takers	0.207	11.934	20.558
Compliers	0.128	12.352	21.118
Never takers	0.150	12.630	21.802

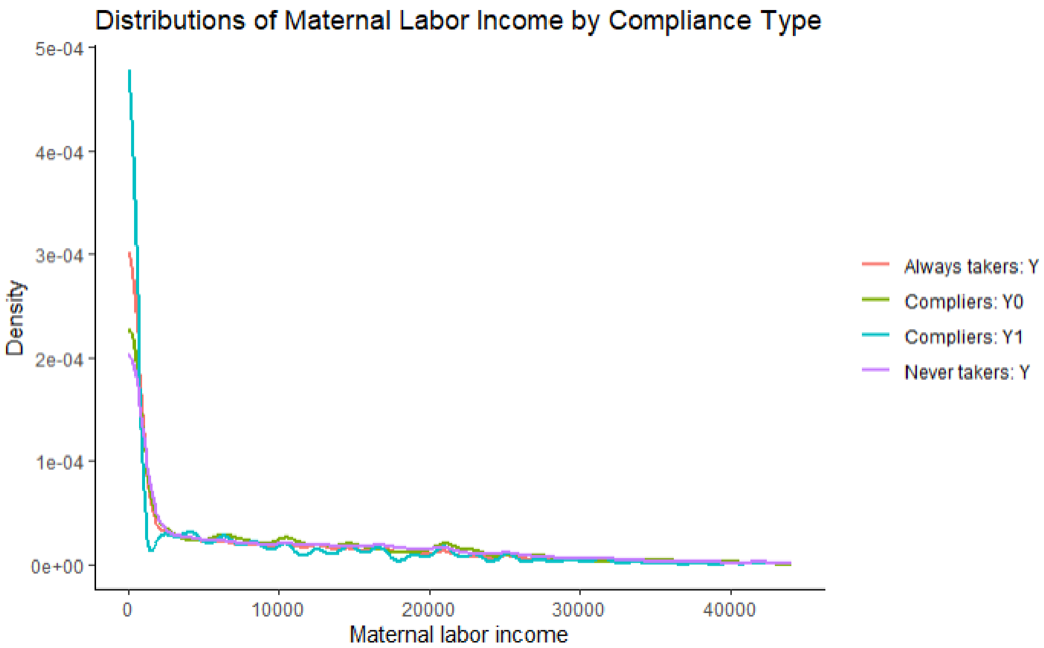

Figure 4: Distributions of Maternal Labor Income by Compliance Type

have the lowest non-white proportion, but lie between always takers and never takers with respect to education and age at first birth, with is consistent with the idea that they are more on the margin in terms of earnings potential and likely to be pushed into having a third child if their first two children are the same gender.

4. Outcome distributions by compliance type

   (a) Plot the distributions of Y_0 and Y_1 for compliers, using maternal labor income as the outcome.

   (b) Plot the distribution of observed Y_i for always takers and for never takers.

   (c) What do the distributions for compliers suggest about the validity of the instrumental variables design?

The four distributions across incomes are plotted in Figure 4 and the distributions across log incomes are plotted in Figure 5. In particular, the recovered distributions for compliers for Y_0 and Y_1 do not show violations of nonnegativity, which must hold if the Imbens-Rubin formulae for complier distributions is valid. If the estimated complier distributions were negative or looked irregular, the IV design would be flawed.

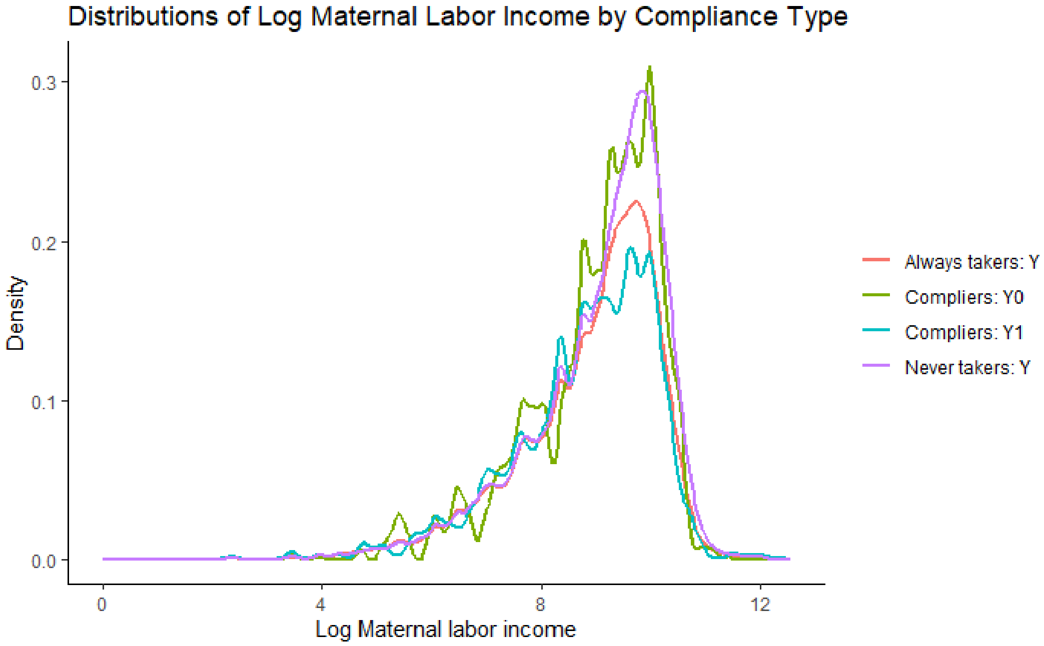

Figure 5: Distributions of Log Maternal Labor Income by Compliance Type

5. Huber–Mellace test

   (a) Implement the Huber and Mellace (2015) test, again using maternal labor income as the outcome.

   (b) You may do so by constructing and reporting the relevant bounds for always takers and never takers.

   (c) You do not need to compute standard errors.

We test both moment inequality conditions. For always takers, we first have:

$$E(Y \mid D=1, Z=0) = 6381.852.$$

The proportion of always takers among individuals with $D=1, Z=1$ is

$$q = \frac{\phi_a}{\phi_a + \phi_c} = 0.868.$$

Thus, we compute the sharp bounds:

$$E(Y \mid D=1, Z=1, Y \leq y_q) = 3058.254, \quad E(Y \mid D=1, Z=1, Y \geq y_{1-q}) = 7272.146,$$

and see the always taker condition is satisfied. For never takers, we have:

$$E(Y \mid D=0, Z=1) = 8537.011.$$

The proportion of never takers among individuals with $D=0, Z=0$ is

$$r = \frac{\phi_n}{\phi_n + \phi_c} = 0.916.$$

We compute the sharp bounds:

$$E(Y \mid D=0, Z=0, Y \leq y_r) = 5877.650, \quad E(Y \mid D=0, Z=0, Y \geq y_{1-r}) = 9229.122,$$

and see the never taker condition is satisfied. Thus, the maternal labor data is consistent with the implications of IV design.